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CIRCULAR ARRAY OF DIPOLES ABOVE A PERFECTLY CONDUCTING CYLINDER

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This report contains an analysis of coupled dipoles near and parallel to an infinite conducting right-circular cylinder, based on integral equations developed by the Fourier transform methods. The computer is used to numerically solve the M nonhomogeneous integral equations, using numerical approximations for the integrals and complex algebra techniques to solve the simultaneous equations. Such an analysis should account for both the space wave with its grating lobe and the creeping wave resonances on the curved surface of the cylinder, and have an impact on the development of optimum methods for

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20. Abstract (Continued)

determining techniques for radiation pattern generation with very low sidelobes, optimum spacing, and a minimum number of array elements.

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Illustrations

Circular Array of Dipoles Above a Perfectly Conducting Cylinder

1. INTRODUCTION

Recent developments in low-inertia phase shifters and electronic switches[1] have aroused interest in large, circular arrays on conducting cylindrical surfaces for electronic agile beam positioning or radar resource optimization, invariant beam characteristics over the entire 360° azimuthal scan sector, and frequency-independent beam pointing direction. A low-sidelobe, high-gain aperture design for a circular array of dipoles surrounding a perfectly conducting cylinder requires a knowledge and control of mutual coupling parameters in the array environment.[2] Carter[3] in 1943 outlined a rigorous solution of Maxwells' equations for a dipole near a long cylinder (see Figure 1), and tabulated formulas for the radiation patterns for three different circular array configurations. Carter used the reciprocity theorem and the known solution for scattering from a cylinder to obtain far-zone

(Received for publication 30 September 1980)

1. Provencher, J.H. (1970) A survey of circular symmetric arrays, Phased Array Antennas (Oliver and Knittel, Artech House, Inc., Dedham, Mass.) page 292.

2. Hessel, A. (1970) Mutual coupling effects in circular arrays on cylindrical surfaces—aperture design implications and analysis, Phased Array Antennas, (Oliver and Knittel, Artech House, Inc., Dedham, Mass.) page 273.

3. Carter, P.S. (1943) Antenna arrays around cylinders, Proceedings of IRE, Vol. 31, December.

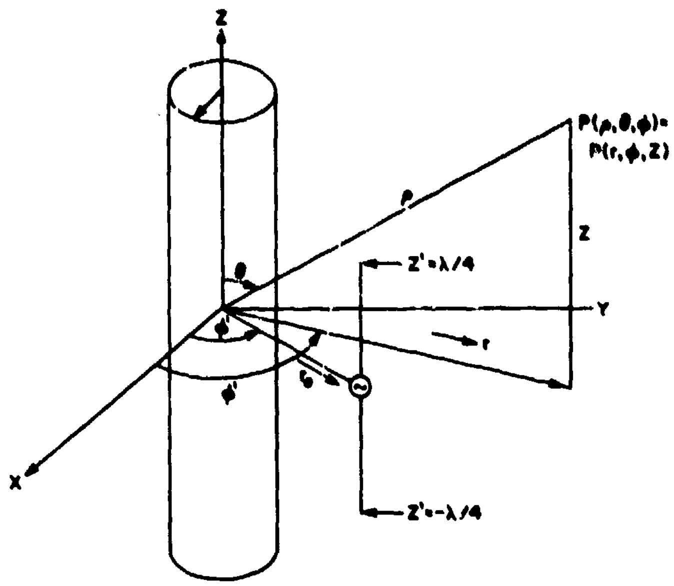

Figure 1. Dipole Near a Long Cylinder

patterns. Lucke[4, 5] used Green's function method to yield expressions for the fields in terms of an integration in the complex plane. The problem was also solved by Harrington[6, 7] by converting a three-dimensional radiation problem having cylindrical boundaries into a two-dimensional problem by applying a Fourier transform with respect to the cylinder axis, Z. Neff[8] also used the Fourier transform method, and by means of a limiting process, obtained an approximate form for a dipole near a finite cylinder.

In this report, the problem of finding the electromagnetic field and admittance of a circular array of dipoles near and parallel to an infinite conducting cylinder

4. Lucke, W. (1949) Electric Dipoles in the Presence of Elliptical and Circular Cylinders, Report No. 1, Project 188, Stanford Research Institute, Stanford, Calif.

5. Lucke, W. (1951) Electric dipoles in the presence of elliptical and circular cylinders, Journal of Applied Physics, Vol. 22, No. 1, January.

6. Harrington, R.F. and LePage, W.R. (1949) A Study of Directional Antennas for DIF Purposes, Report 1, Department of Electrical Engineering, Syracuse University, Syracuse, NY, September.

7. Harrington, R.F. (1961) Time-Harmonic Electromagnetic Fields, McGraw-Hill, New York, NY, Chapter 3.

8. Neff, H.P., Hickman, C.E., and Tillman, J.D. (1964) Circular Arrays Around Cylinders, Report No. 7, Department of Electrical Engineering, University of Tennessee, Contract AF19(628)-288, June.

has been mathematically formulated into integral equations via the Fourier transform method, which relates the illumination function's voltages to the resultant current distribution due to the mutual coupling. We use the computer to numerically solve the m nonhomogeneous integral equations, using numerical approximations for the integrals and complex matrix techniques to solve the simultaneous equations. Such an analysis should account for both the space wave with its grating lobe and the creeping wave resonances on the curved surface, and have an impact on the development of optimum methods for determining techniques for radiation pattern generation with very low sidelobes, optimum spacing and a minimum number of array elements (see Figure 2). We shall investigate the effects of mutual coupling for a circular array located a quarter of a wavelength above a perfectly conducting cylinder having a radius of two wavelengths. This simple case was chosen to easily investigate the m simultaneous integral equations and to lay the foundation for future investigations into the more practical case of a perfectly conducting cylinder with radius of approximately 10 wavelengths.

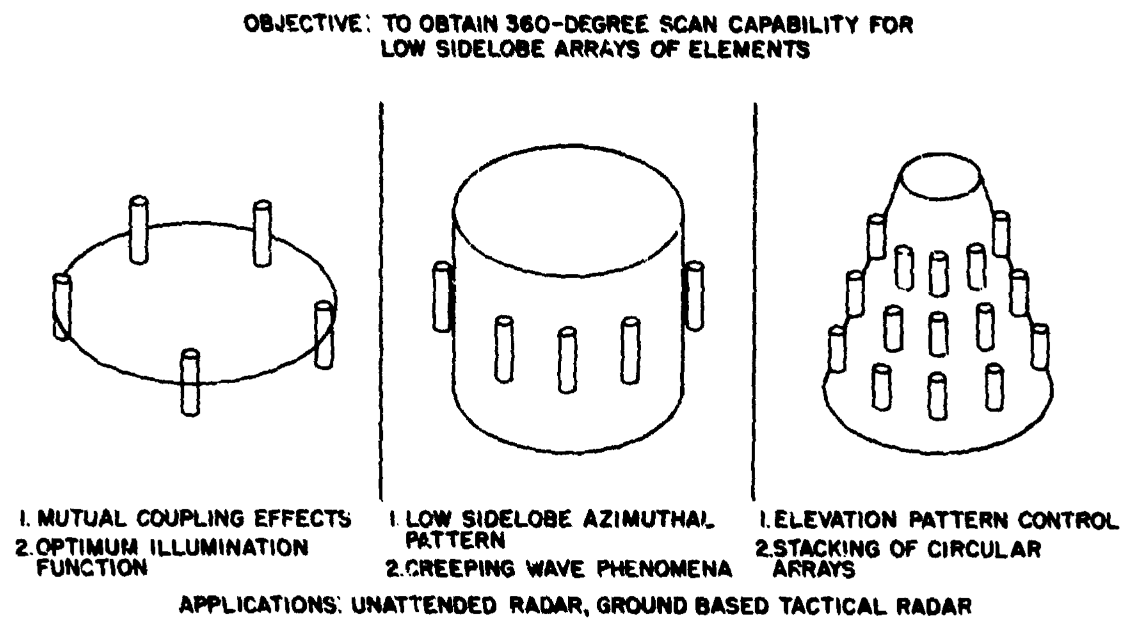

Figure 2. Circular Arrays Around Cylinder, Optimization Approach

2. MATHEMATICAL ANALYSIS

The circular antenna array to be considered (see Figure 3) consists of identical, parallel, cylindrical dipoles equally spaced around the circumference of a circle, which is concentric to the perfectly conducting cylinder which the array of dipoles surrounds. Only center-fed dipoles with a half-length $\lambda/4$ and a radius b will be used as elements. The axial center of each dipole is perpendicular to the plane of the circle. Each of the elements of the array is thus in the same geometrical environment. The radius of the perfectly conducting cylinder is a, and the radius of the circular array of dipoles is r_o (see Figure 4). The expression for the vector potential A (r, ϕ, z) has been developed for a circular array of m dipoles located about a perfectly conducting cylinder.[9]

For a current density $\vec{j}$ located above a perfectly conducting cylinder (see Figure 5),

$$\nabla^2 \vec{A} + k^2 \vec{A} = -\mu_o \vec{j} \quad . \tag{1}$$

For a unit dipole at (r_o, ϕ_o, z_o),

$$\vec{j} = \hat{z} \; \frac{\delta(r - r_o)\delta(\phi - \phi_o)\delta(z - z_o)}{r} \quad . \tag{2}$$

Therefore, we need only be concerned with the A_z component:

$$f(z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\alpha, r, \phi)\, e^{i\alpha z}\, d\alpha \tag{3}$$

where

$$F(\alpha, r, \phi) = \sum_{m=-\infty}^{\infty} a_m(\alpha, r)\, e^{im\phi} \quad . \tag{4}$$

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9. Fante, Dr. Ronald L. (private communication).

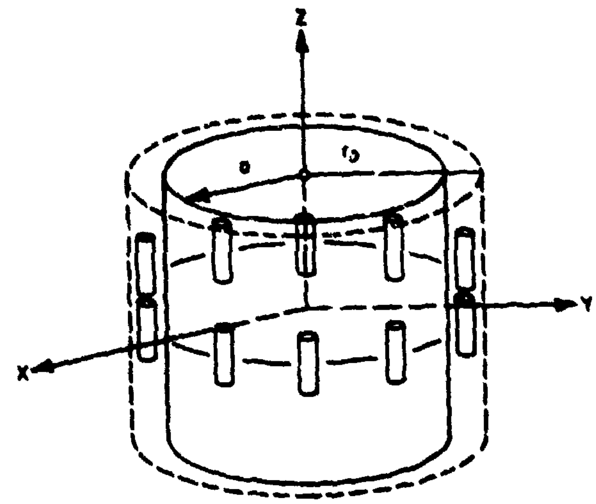


Figure 3. Perspective View of Array of Cylindrical Dipoles Surrounding a Perfectly Conducting Cylinder

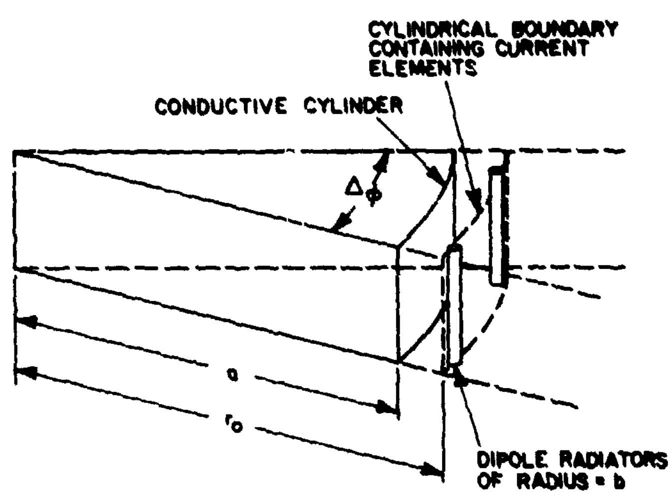


Figure 4. Perspective View of Dipole, Conductive Cylinder Geometry

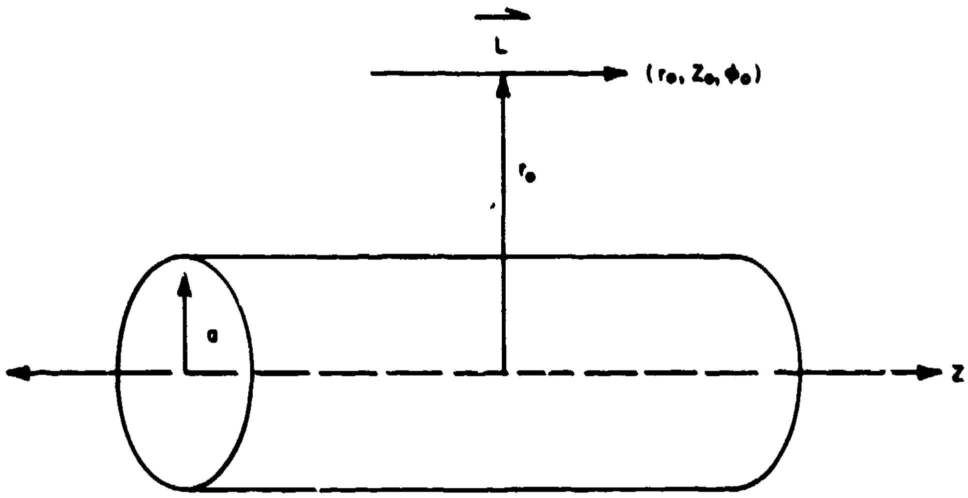


Figure 5. Geometry for the Development of Integral Equations for a Circular Array Around a Perfectly Conducting Cylinder

From Eqs. (3) and (4) we get

$$A_z = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\alpha\, e^{i\alpha z} \sum_{m=-\infty}^{\infty} a_m(\alpha, r)\, e^{im\phi} \quad . \tag{5}$$

In cylindrical coordinates, the laplacian is

$$\nabla^2 A_z = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial A_z}{\partial r} \right) + \frac{1}{r^2} \left(\frac{\partial^2 A_z}{\partial \phi^2} \right) + \frac{\partial^2 A_z}{\partial z^2} \quad . \tag{6}$$

Substitute Eqs. (6), (5), and (2) into Eq. (1) to yield

$$\frac{1}{2\pi} \sum_{m=-\infty}^{\infty} e^{im\phi} \int_{-\infty}^{\infty} d\alpha\, e^{i\alpha z} \left\{ \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial a_m}{\partial r} \right) + \left(k^2 - \alpha^2 - \frac{m^2}{r^2} \right) a_m \right\} =$$

$$- \mu_o \frac{\delta(r - r_o)\delta(\phi - \phi_o)\delta(z - z_o)}{r} \quad . \tag{7}$$

Multiply both sides of Eq. (7) by $e^{in\phi}\, e^{i\alpha' z}$ then integrate

$$\frac{1}{2\pi} \sum_{m=-\infty}^{\infty} \int_{-\infty}^{\infty} d\alpha \left\{ \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial a_m}{\partial r} \right) + \left(k^2 - \alpha^2 - \frac{m^2}{r^2} \right) a_m \right\} \int_{-\pi}^{\pi} d\phi\, e^{i(m-n)\phi}$$

$$\times \int_{-\infty}^{\infty} dz\, e^{(\alpha-\alpha')z} = \frac{-\mu_o \delta(r - r_o)}{r} \int_{-\pi}^{\pi} d\phi\, e^{in\phi}\, \delta(\phi - \phi_o) \int_{-\infty}^{\infty} dz\, \delta(z - z_o)\, e^{-i\alpha' z} \tag{8}$$

Use the relations

$$\int_{-\pi}^{\pi} e^{i(m-n)\phi}\, d\phi = \begin{cases} 0 \text{ if } m \neq n \\ 2\pi \text{ if } m = n \end{cases}$$

$$\int_{-\infty}^{\infty} e^{i(\alpha-\alpha')z}\, dz = \begin{cases} 0 \text{ if } m \neq n \\ 2\pi \text{ if } m = n \end{cases}$$

then Eq. (8) reduces to

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial a_n}{\partial r} \right) + \left(k^2 - \alpha^2 - \frac{n^2}{r^2} \right) a_n = \frac{-\mu_o}{2\pi r} \delta(r - r_o)\, e^{-in\phi}\, e^{-i\alpha' z} \tag{9}$$

For $r > r_o$

$$a_n = B_n\, e^{-in\phi_o}\, e^{-i\alpha' z_o}\, H_n \left(\sqrt{k^2 - \alpha'^2}\, r\right) \tag{10}$$

For $r < r_o$

$$a_n = e^{-in\phi_o}\, e^{-i\alpha' z_o} \left\{ C_n J_n \left(\sqrt{k^2 - \alpha'^2}\, r\right) + D_n H_n \left(\sqrt{k^2 - \alpha'^2}\, r\right) \right\} \tag{11}$$

Next we derive continuity conditions at $r = r_o$ from Eq. (9). Integrate Eq. (9) from $r_o - \epsilon$ to $r_o + \epsilon$ and let $\epsilon \to 0$.

$$\int_{r_o-\epsilon}^{r_o+\epsilon} \frac{\partial}{\partial r}\left(r\,\frac{\partial a_n}{\partial r}\right) dr + \int_{r_o-\epsilon}^{r_o+\epsilon} r\left(k^2 - \alpha^2 - \frac{n^2}{r^2}\right) a_n\, dr =$$

$$\frac{-\mu_o\, e^{-in\phi_o}\, e^{-i\alpha' z_o}}{2\pi} \int_{r_o-\epsilon}^{r_o+\epsilon} \delta(r - r_o)\, dr$$

therefore

$$\left(\frac{\partial a_n}{\partial r}\right)_{r_o+\epsilon} - \left(\frac{\partial a_n}{\partial r}\right)_{r_o-\epsilon} = \frac{-\mu_o}{2\pi r_o}\, e^{-in\phi_o}\, e^{-i\alpha' z_o} \quad . \tag{12}$$

Substitute Eqs. (10) and (11) into Eq. (12) to yield

$$B_n\left[\frac{\partial}{\partial r} H_n(\sqrt{k^2 - \alpha^2}\, r)\right]_{r_o} - C_n\left[\frac{\partial}{\partial r} J_n(\sqrt{k^2 - \alpha^2}\, r)\right]_{r_o} -$$

$$- D_n\left[\frac{\partial}{\partial r} H_n(\sqrt{k^2 - \alpha^2}\, r)\right]_{r_o} = \frac{-\mu_o}{2\pi r_o} \tag{13}$$

define

$$\frac{\partial}{\partial x} H_n(x) = H_n'(x)$$

$$x \equiv \sqrt{k^2 - \alpha^2}\, r_o \quad .$$

Then from Eq. (13) we get

$$(B_n - D_n)H_n'(x) - C_n J_n'(x) = \frac{-\mu_o}{2\pi x} \quad . \tag{14}$$

Equation (14) is the first solution between B_n, C_n, and D_n. Since A_z must be continuous at $r = r_o$,

$$B_n H_n(x) = C_n J_n(x) + D_n H_n(x)$$

or

$$B_n - D_n = \frac{C_n J_n(x)}{H_n(x)} \quad . \tag{15}$$

Substitute Eq. (15) into Eq. (14) to yield

$$C_n \left\{ \frac{J_n(x)\, H_n'(x)}{H_n(x)} - J_n'(x) \right\} = \frac{-\mu_o}{2\pi x} \quad .$$

Define $\Delta = J_n(x)\, H_n'(y) - J_n'(x)\, H_n(x)$ (Wronskian), then

$$C_n = \frac{-\mu_o H_n(x)}{2\pi \times \Delta} \quad . \tag{16}$$

Also from Eq. (15),

$$B_n - D_n = \frac{-\mu_o J_n(x)}{2\pi \times \Delta} \quad . \tag{17}$$

Now we need to satisfy the boundary conditions at the surface $r = a$ of the cylinder, which is that the electric field tangential on its cylindrical surface must be zero.

$$\vec{E} = \frac{1}{jw\epsilon_o \mu_o} \nabla \times \nabla \times \vec{A}$$

$$\nabla \times \vec{A} = \hat{r}\, \frac{1}{r} \left(\frac{\partial A_z}{\partial \phi} \right) - \hat{\phi}\, \frac{\partial A_z}{\partial r}$$

$$\nabla \times \nabla \times \vec{A} = \hat{r}\, \frac{\partial^2 A_z}{\partial r \partial z} + \hat{\phi}\, \frac{1}{r} \frac{\partial^2 A_z}{\partial z \partial \phi} + \hat{z} \left[-\frac{1}{r} \frac{\partial}{\partial r} \left(r\, \frac{\partial A_z}{\partial r} \right) - \frac{1}{r^2} \frac{\partial^2 A_z}{\partial \phi^2} \right]$$

therefore

$$\left[\frac{\partial^2 A_z}{\partial z \partial \phi} \right]_{r=a} = 0 \tag{18}$$

and

$$\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\,\frac{\partial A_z}{\partial r}\right) + \frac{1}{r^2}\,\frac{\partial^2 A_z}{\partial \phi^2}\right]_{r=a} = 0 \tag{19}$$

But from the wave equation

$$\frac{1}{r}\,\partial r\left(r\,\frac{\partial A_z}{\partial r}\right) + \frac{1}{r^2}\left(\frac{\partial^2 A_z}{\partial \phi^2}\right) + \frac{\partial^2 A_z}{\partial z^2} + k^2 A_z = 0$$

Therefore, Eq. (19) is equivalent to

$$\frac{\partial^2 A_z}{\partial z^2} + k^2 A_z = 0 \tag{20}$$

Upon substitution of Eq. (5) in Eqs. (18) and (20), we get

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} d\alpha(i\alpha)(im)\, e^{i\alpha z} \sum_{m=-\infty}^{\infty} a_m\, e^{im\phi} = 0 \tag{21}$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} d\alpha(k^2 - \alpha^2)\, e^{i\alpha z} \sum_{m=-\infty}^{\infty} a_m\, e^{im\phi} = 0 \tag{22}$$

where

$$a_n = e^{-in\phi_o}\, e^{-i\alpha z_o}\left\{C_n J_n(\sqrt{k^2 - \alpha^2}\, a) + D_n H_n(\sqrt{k^2 - \alpha^2}\, a)\right\} \tag{23}$$

Equations (21) and (22) are both equivalent to requiring that $A_m = 0$ at $r = a$, therefore

$$C_n J_n(y) + D_n H_n(y) = 0$$

where

$$y = \sqrt{k^2 - \alpha^2}\, a$$

$$D_n = \frac{-C_n J_n(y)}{H_n(y)} \tag{24}$$

Upon substituting Eq. (16) into Eq. (24) and rearranging terms we get

$$D_n = \frac{\mu_o H_n(x)}{2\pi \times \Delta} \left(\frac{J_n(y)}{H_n(y)} \right) \tag{25}$$

Using Eq. (25) in Eq. (17) we get

$$B_n = \frac{-\mu_o}{2\pi \times \Delta H_n(y)} \left\{ J_n(x) H_n(y) - H_n(x) J_n(y) \right\} \tag{26}$$

Since $\pi \times \Delta = 2i$ we get

$$B_n = \frac{i\mu_o}{4H_n(y)} \left\{ J_n(x) H_n(y) - H_n(x) J_n(y) \right\} \tag{27}$$

$$C_n = \frac{i\mu_o H_n(x)}{4} \tag{28}$$

$$D_n = \frac{\mu_o H_n(x)}{4i} \frac{J_n(y)}{H_n(y)} \tag{29}$$

where

$x = \sqrt{k^2 - \alpha^2}\, r_o$

$y = \sqrt{k^2 - \alpha^2}\, a$

r_o = radius at which dipole is located

a = radius of cylinder.

Now that $B_n(\alpha)$, $C_n(\alpha)$, and $D_n(\alpha)$ are known, the Green's function is for $r \le r_o$

$$G_1 = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\alpha\, e^{i\alpha(z-z_o)} \sum_{n=-\infty}^{\infty} e^{in(\phi-\phi_o)} \Big[C_n(\alpha) J_n(\sqrt{k^2-\alpha^2}\, r) +$$

$$D_n(\alpha) H_n(\sqrt{k^2-\alpha^2}\, r)\Big]$$

and for $r \ge r_o$

$$G_2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i\alpha(z-z_o)} d\alpha \sum_{n=-\infty}^{\infty} e^{in(\phi-\phi_o)} B_n(\alpha) H_n(\sqrt{k^2-\alpha^2}\, r)$$

Therefore for any arbitrary axial current density $j\,(r, \phi, z)$

$$A_z = \iiint dv\, j \begin{Bmatrix} G_1 & \text{if} & r \le r_o \\ G_2 & \text{if} & r \ge r_o \end{Bmatrix} \tag{30}$$

where

dv = volume of current density.

For a thin current filament we have the relationship $I \simeq \pi R_1^2 j$ where R_1 is the radius of the current element. Then $dv \simeq \pi R_1^2 dz_o$ and Eq. (30) becomes

$$A(r, \phi, z) = \sum_{m=1}^{m} \int_{-L/2}^{L/2} dz_o\, I(z_o) \begin{Bmatrix} G_1(r, \phi, z) & \text{if} & r \le r_o \\ G_2(r, \phi, z) & \text{if} & r \ge r_o \end{Bmatrix} \tag{31}$$

the expression for the vector potential for a circular array of m dipoles located about a perfectly conducting cylinder.

We now need to relate the currents on the circular array elements to the base voltages to determine the self and mutual admittances of the circular array around a perfectly conducting cylinder. It can be shown[10] that the vector potential at any point on the surface of antenna K (see Figure 6) is given by

10. Tillman, James D. (1966) The Theory and Design of Circular Antenna Arrays, Chapter 1, The University of Tennessee, Engineering Experiment Station.

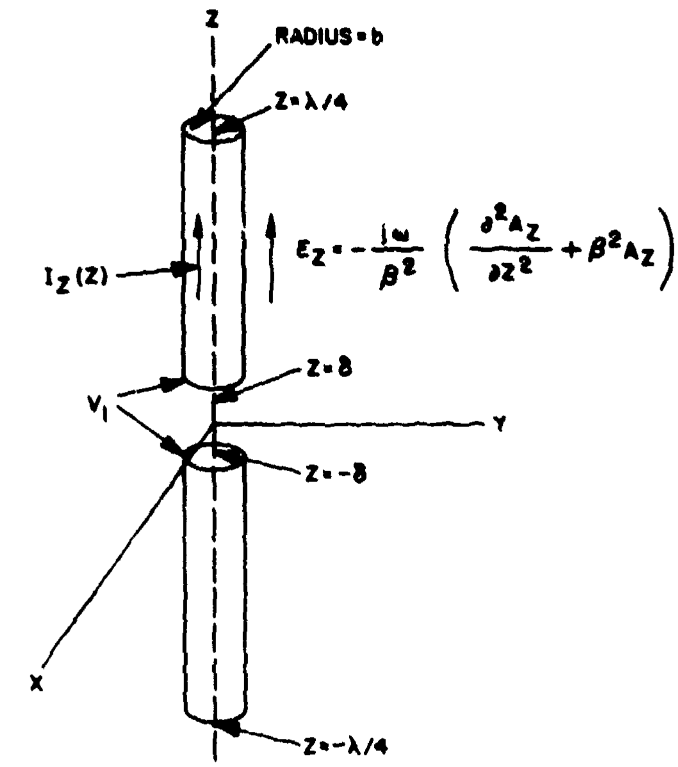


Figure 6. Cylindrical Dipole, Showing the Method of Applying the Boundary Condition $E_z = 0$ at $r_d = b$

$$\left[A_z(z_o)\right]_{r_d = b} = \frac{-j}{c}\left[C_k \cos \beta_z + \frac{V_K}{2} \sin \beta_z\right] \tag{32}$$

where b is the radius of the dipole, $-h \le z \le h$, $h = \frac{L}{2}$, V_K is the base voltage of the K element, and $\beta = 2\pi/\lambda$. Equating Eq. (32) to Eq. (31) we get

$$\sum_{m=1}^{m} \int_{-L/2}^{L/2} dz_o\, I_m(z_o) \begin{cases} G_1(r, \phi, z) & \text{if} \quad r \le r_o \\ G_2(r, \phi, z) & \text{if} \quad r \ge r_o \end{cases} = \frac{-j}{c}\left[C_k \cos \beta_z + \frac{V_K}{2} \sin \beta_z\right]$$

for $r \geq r_o$ we get

$$\sum_{m=1}^{m} \int_{-L/2}^{L/2} dz_o\, I_m(z_o) \left\{ \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i\alpha(z-z_o)}\, d\alpha \sum_{n=-\infty}^{\infty} e^{in(\phi_k + b/r_o - \phi_m)} \right.$$

$$\times B_n(\alpha) H_n(\sqrt{k^2 - \alpha^2}\, r) = \frac{-j}{c} \left[C_k \cos \beta_z + \frac{V_K}{2} \sin \beta |z| \right] \tag{33}$$

where the K^{th} dipole is at (r, ϕ, z). Thus, for an m-dipole circular array, we have m simultaneous equations to solve. Let $z = \pi/2\beta$, then Eq. (33) becomes

$$\frac{1}{2\pi} \sum_{m=1}^{m} \int_{-L/2}^{L/2} dz_o\, I_m(z_o) \int_{-\infty}^{\infty} e^{i\alpha(\pi/2\beta - z_o)}\, d\alpha \sum_{n=-\infty}^{\infty} e^{in(\phi_k + b/r_o - \phi_m)}$$

$$\times B_n(\alpha) H_n(\sqrt{k^2 - \alpha^2}\, r) = \frac{-j\, V_K}{2C} \tag{34}$$

Substitute Eq. (27) into Eq. (34) and let $I_m(z_o) = A_m \cos \frac{\pi z_o}{L}$ then,

$$\frac{1}{2\pi} \sum_{m=1}^{m} A_m \int_{-L/2}^{L/2} dz_o \cos \frac{\pi z_o}{L} \int_{-\infty}^{\infty} d\alpha\, e^{i\alpha(\pi/2\beta - z_o)} \sum_{n=-\infty}^{\infty} e^{in(\phi_k + b/r_o - \phi_m)}$$

$$\times \left[\frac{J_n(x) H_n(y) - H_n(x) J_n(y)}{H_n(y)} \cdot H_n(\sqrt{k^2 - \alpha^2}\, r) \right] = \frac{-4\pi V_K}{\mu_o C} \tag{35}$$

Since all dipoles are equidistant from the perfectly conducting cylinder, $r = r_o$ then,

$$H_n(\sqrt{k^2 - \alpha^2}\, r) = H_n(\sqrt{k^2 - \alpha^2}\, r_o) = H_n(x)$$

Thus Eq. (35) becomes

$$\sum_{m=1}^{m} A_m \int_{-L/2}^{L/2} dz_o \cos\frac{\pi z_o}{L} \sum_{n=-\infty}^{\infty} e^{in(\phi_k + b/r_o - \phi_m)} \int_{-\infty}^{\infty} d\alpha\, e^{i\alpha(\pi/2\beta - z_o)}$$

$$\times \left[J_n(x)\, H_n(x) - \frac{H_n^2(x)\, J_n(y)}{H_n(y)} \right] = \frac{-4\pi V_K}{\mu_o C}$$

or

$$\sum_{m=1}^{m} A_m \sum_{n=-\infty}^{\infty} e^{in(\phi_k + b/r_o - \phi_m)} \int_{-\infty}^{\infty} d\alpha\, e^{i\alpha(\frac{\pi}{2\beta})}\, g(\alpha)$$

$$\times \left[J_n(x)\, H_n(x) - \frac{H_n^2(x)\, J_n(y)}{H_n(y)} \right] = \frac{-4\pi V_K}{\mu_o C} \qquad (36)$$

where

$$g(\alpha) = \int_{-L/2}^{L/2} dz_o \cos\frac{\pi z_c}{L}\, e^{i\alpha z_o}$$

Equation (36) reduces to

$$\sum_{m=1}^{m} A_m \sum_{n=-\infty}^{\infty} e^{in(\phi_k + b/r_o - \phi_m)} \int_{-\infty}^{\infty} d\alpha\, \frac{\cos\frac{L\alpha}{2}}{\frac{\pi^2}{L^2} - \alpha^2}\, e^{i\alpha(\pi/2\beta)}$$

$$\times \left[J_n(x)\, H_n(x) - \frac{J_n(y)\, H_n^2(x)}{H_n(y)} \right] = \frac{-2V_K L}{\mu_o C}$$

which from symmetry becomes

$$\sum_{m=1}^{m} A_m \sum_{n=0}^{\infty} e^{in(\phi_k+b/r_o-\phi_m)} \int_0^{\infty} d\alpha \, \frac{\cos\frac{L\alpha}{2}}{\frac{\pi^2}{L^2}-\alpha^2} \cos\frac{\alpha\lambda}{4}$$

$$\times \left[J_n(x) H_n(x) - \frac{H_n^2(x) J_n(y)}{H_n(y)} \right] = \frac{-V_K L}{2\mu_o C} \qquad (37)$$

2.1 Self and Mutual Admittance

Now, let

$$T_{km}(\alpha) = \sum_{n=0}^{\infty} e^{in(\phi_k+b/r_o-\phi_m)} \int_0^{\infty} d\alpha \, \frac{\cos\frac{L\alpha}{2}}{\frac{\pi^2}{L^2}-\alpha^2} \cos\frac{\alpha\lambda}{4}$$

$$\times \left[J_n(x) H_n(x) - \frac{H_n^2(x) J_n(y)}{H_n(y)} \right]$$

then

$$\sum_{m=1}^{m} A_m T_{km}(\alpha) = \frac{-V_K L}{\mu_o C}$$

Now we shall consider the M integral equations by taking a look at the equations yielded for each base voltage V_k (K = 1, ... m) simultaneously. We get

$$\begin{array}{l} A_1 T_{11} + A_2 T_{12} + \dots A_m T_{1m} = \frac{-L}{\mu_o C} V_1 \\ A_1 T_{21} + A_2 T_{22} + \dots A_m T_{2m} = \frac{-L}{\mu_o C} V_2 \\ \quad \cdot \qquad \cdot \qquad \cdot \qquad \cdot \\ \quad \cdot \qquad \cdot \qquad \cdot \qquad \cdot \\ \quad \cdot \qquad \cdot \qquad \cdot \qquad \cdot \\ A_1 T_{m1} + A_2 T_{m2} + \dots A_m T_{mm} = \frac{-L}{\mu_o C} V_m \end{array} \qquad (38)$$

In a matrix notation, Eq. (38) is equal to

$$\begin{bmatrix} T_{11} & T_{12} & \cdots & T_{1m} \\ T_{12} & T_{22} & \cdots & T_{2m} \\ \cdot & & \cdots & \\ \cdot & & \cdots & \\ \cdot & & \cdots & \\ T_{m1} & T_{m2} & \cdots & T_{mm} \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ \cdot \\ \cdot \\ \cdot \\ A_m \end{bmatrix} = \begin{bmatrix} \tilde{V}_1 \\ \tilde{V}_2 \\ \cdot \\ \cdot \\ \cdot \\ \tilde{V}_m \end{bmatrix} \tag{39}$$

where

$$\tilde{V}_k = \frac{-L}{\mu_o C} V_K \quad .$$

Therefore,

$$\tilde{T} \tilde{A} = \tilde{\vec{V}} \quad .$$

Given values for V_1, V_2, ..., V_m and having numerically approximated the integrals T_{km} ($m = 1, \ldots, m$; $K = 1, \ldots, m$), we compute the values for the A_m's by factoring the matrix T into the L - U decomposition of a row-wise permutation of T and solving the systems of Eq. (37). Once the values for the A_m's have been found, we can then determine the mutual and self admittances of the circular array. Equation (33) gives the relationship between the currents and voltages of the array elements. For an m-element array,

$$\begin{array}{l} Y_{11}V_1 + Y_{12}V_2 + \cdots + Y_{1k}V_k + \cdots + Y_{1m}V_m = I_1 \\ Y_{21}V_1 + Y_{22}V_2 + \cdots + Y_{2k}V_k + \cdots + Y_{2m}V_m = I_2 \\ \quad \cdot \qquad \cdots \qquad \cdot \qquad \cdots \qquad \cdot \\ \quad \cdot \qquad \cdots \qquad \cdot \qquad \cdots \qquad \cdot \\ \quad \cdot \qquad \cdots \qquad \cdot \qquad \cdots \qquad \cdot \\ Y_{m}V_1 + Y_{m2}V_2 \ \cdots + Y_{mk}V_k + \cdots + Y_{mm}V_m = I_m \ . \end{array} \tag{40}$$

where Y_{11}, Y_{22}, ... Y_{kk}, ... Y_{mm} are the self admittance of the respective element, Y_{ab} is the mutual admittance between a and b, V_K is the applied voltage on the K^{th} element, and I_K is the current in the K^{th} element.

For an array with a single active element, which for simplicity's sake we let be element no. 1, every V_K where $K = 1$ is equal to zero. Thus, Eq. (16) becomes

$$
\begin{aligned}
I_1 &= Y_{11}V_1 \\
I_2 &= Y_{21}V_1 \\
I_3 &= Y_{31}V_1 \\
&\cdot \\
&\cdot \\
&\cdot \\
&\cdot \\
I_m &= Y_{m1}V_1 \quad .
\end{aligned}
\tag{41}
$$

We let $V_1 = 1$ and A_K is the magnitude and phase of the current on element K. Therefore,

$$
\begin{aligned}
A_1 &= Y_{11} \\
A_2 &= Y_{21} \\
&\cdot \\
&\cdot \\
&\cdot \\
&\cdot \\
A_m &= Y_{m1} \quad .
\end{aligned}
\tag{42}
$$

Thus, the coefficient A_1 is the self admittance of the active element no. 1 and the coefficient A_K, where K is unequal to one, is the mutual admittance between the first and K^{th} element.

2.2 Numerical Results

The simple case of $a = 2\lambda$ and $r_o = a + \lambda/4$ (see Figure 7) was investigated and the mutual and self admittances were determined through use of computer programs to numerically solve the m nonhomogeneous simultaneous equations (Eq. (37)). Equation (37) becomes

$$
\sum_{m=1}^{m} A_m \sum_{n=0}^{\infty} e^{in(\phi_k + b/r_o - \phi_m)} \, \text{BESS}(n) = \frac{-V_K L}{2\mu_o C} \tag{43}
$$

where

$$BESS(n) = \int_0^k d\alpha\, Q(\alpha)\, ReFS(n, \alpha) + \int_{k+\epsilon}^{\infty} d\alpha\, Q(\alpha)\, ReFB(n, \alpha)$$

$$+ i \left\{ \int_0^k d\alpha\, Q(\alpha)\, ImFS(n, \alpha) + \int_{k+\epsilon}^{\infty} d\alpha\, Q(\alpha)\, ImFB(n, \alpha) \right\} \tag{44}$$

$$Q(\alpha) = \frac{\cos \frac{L\alpha}{2}}{\frac{\pi^2}{L^2} - \alpha^2} \cos \frac{\alpha \lambda}{4} \tag{45}$$

$$ReFS(n,\alpha) = J_n(x)J_n(x) - \frac{J_n^2(x)J_n^2(y) - Y_n^2(x)J_n^2(y) + 2J_n(x)Y_n(x)J_n(y)Y_n(y)}{J_n^2(y) + Y_n^2(y)} \tag{46}$$

$$ReFB(n,\alpha) = I_n(x)K_n(x) \tag{47}$$

$$ImFS(n,\alpha) = J_n(x)Y_n(x) - \frac{2J_n(x)Y_n(x)J_n^2(y) - J_n^2(x)J_n(y)Y_n(y) + Y_n^2(x)J_n(y)Y_n(y)}{J_n^2(y) + Y_n^2(y)} \tag{48}$$

$$ImFB(n,\alpha) = \frac{2}{\pi} \left\{ \frac{K_n^2(x)I_n(y)}{K_n(y)} \right\} \tag{49}$$

For Eq. (43) to be solvable, the expression

$$\sum_{n=0}^{\infty} e^{in(\phi_k + b/r_o - \phi_m)}\, BESS(n) \tag{50}$$

must converge to zero for large n, otherwise, the appropriate coefficients for each A_m (m = 1, 2, ... m) cannot be determined. Let

$$SR(n) = \int Q(\alpha) ReFS(n,\alpha)\, d\alpha$$

$$SI(n) = \int Q(\alpha) ImFS(n,\alpha)\, d\alpha$$

$$BR(n) = \int Q(\alpha) \mathrm{Re} FB(n,\alpha)\, d\alpha$$

$$BI(n) = \int Q(\alpha) \mathrm{Im} FB(n,\alpha)\, d\alpha$$

The contributions to BESS(n)[11] of SR and BI rapidly diminish for large order n. BR and SI decrease very slowly in absolute value for large order n. Figures 8, 9, 10, and 11 show the values of SR, SI, BR and BI respectively for order n = 0 to 700. BR and SI approach zero very slowly for large order n, while SR and SI converge rapidly to zero for large n. Figure 12 shows the value BSUM(n) where

$$BSUM(n) = \sqrt{(SR(n) + BR(n))^2 + (SI(n) + BI(n))^2} = |BESS(n)|$$

and $\phi_k = \phi_m = 0^\circ$ (worst case). BSUM is shown to slowly converge to zero for large order n. Due to time limitation in computer usage, n = 0 to 700 was chosen; however, for more reliable results, n = 0 to 2000 may be required, especially for the worst case $\phi_k = \phi_m = 0^\circ$.

Once the BESS(n) were determined for n = 0 to 700, these values were used in Eq. (50) to determine the coefficients for the A_m's in the set of simultaneous non-homogeneous equations (Eq. (37)). Each resultant matrix was found to be of the form

$$\begin{bmatrix} T_1 & T_m & \cdots & T_2 \\ T_2 & T_1 & \cdots & T_3 \\ \cdot & & \cdots & \\ \cdot & & \cdots & \\ \cdot & & \cdots & \\ T_m & T_{m-1} & \cdots & T_1 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ \cdot \\ A_m \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ \cdot \\ V_m \end{bmatrix} . \qquad (51)$$

Thus, due to symmetry, Eq. (37) need only be evaluated once for any m = 1, 2, ... m to determine all coefficients for the A_m's in the equation.

Results of the self and mutual admittances for a circular array of cylindrical dipoles a quarter of a wavelength above a perfectly conducting cylinder of radius $r = 2\lambda$ were calculated. See Figure 12 for array sizes of m = 2 to 22. Figure 13 shows the self admittance for the active element, Figure 14 shows the mutual admittance between dipole no. 1 and dipole no. m. Due to symmetry, these two should be the same. The inconsistency may be the result of not taking large enough

11. Amos, V. E., and Daniel, S. L. (1977) AMOSLIB, A Special Library Version, Sandia Laboratories (SAND 77-1390).

order n in Eq. (50) to have BESS(n) approximately equal to zero. Figure 15 shows the mutual admittance between dipole no. 1 and the adjacent dipole no. m. Figure 16 shows the relative currents for the dipoles for arrays of size m = 2, 4, 7, 10, 13, 16. Once again, for m ≥ 12, asymmetries occur which may be due to not taking high enough order n for the evaluation of BESS(n) in Eq. (50) for the determination of the coefficients for the A_m's.

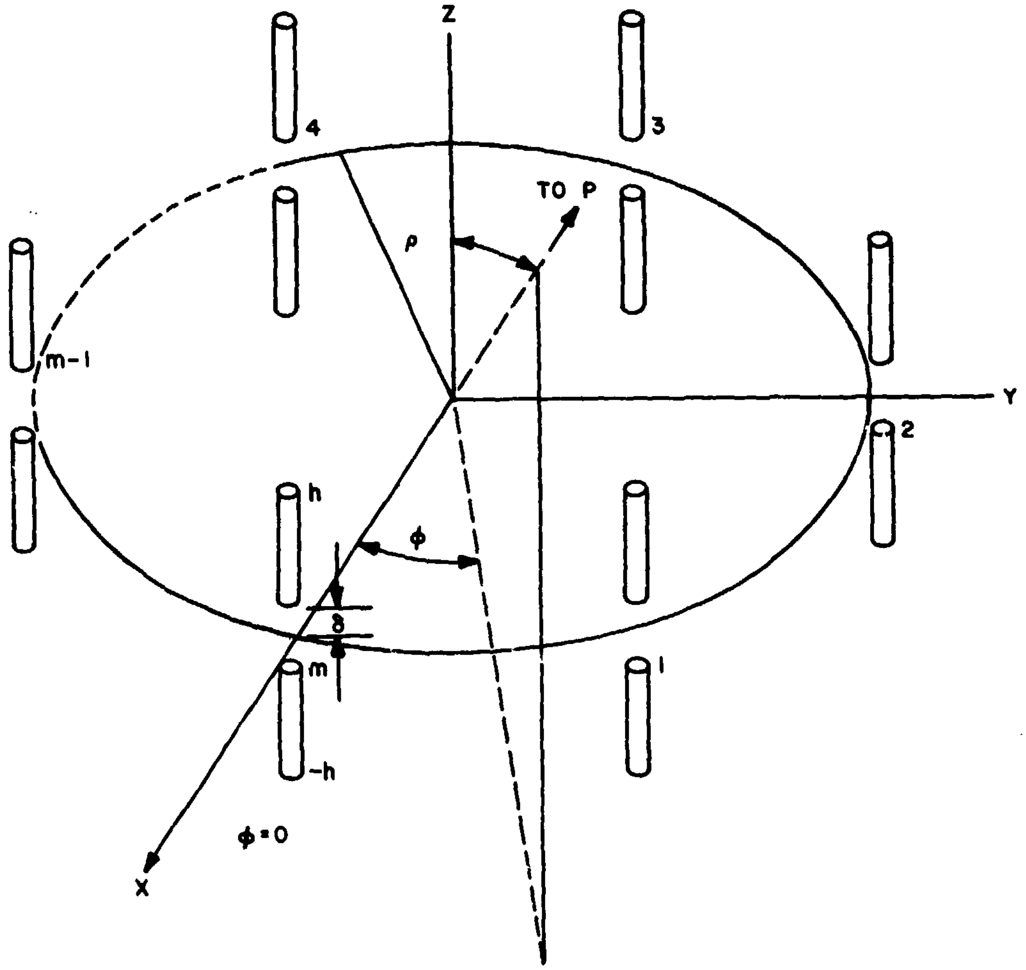


Figure 7. Array of Cylindrical Dipoles About a Perfectly Conducting Cylinder, Simple Case, $a = 2\lambda$, $\rho = r_o = a + \lambda/4$

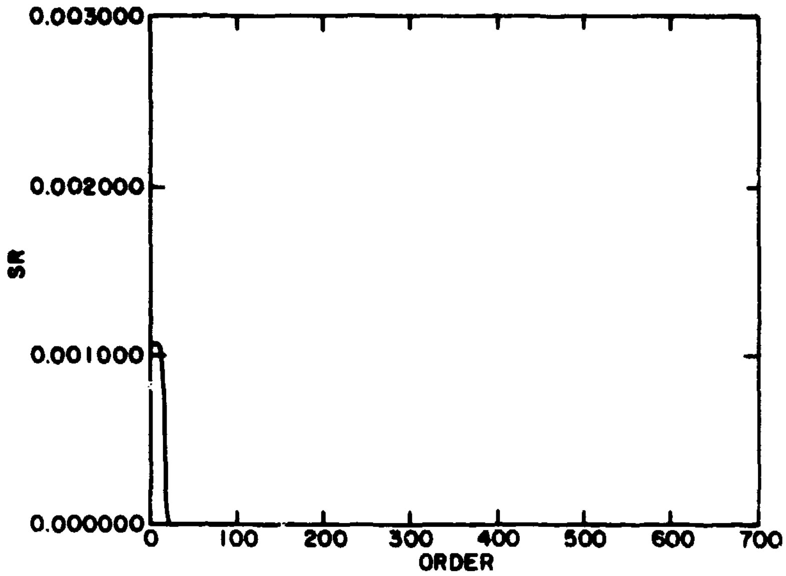


Figure 8. SR(n) = $\int Q(\alpha)$ReFS(N, α) for n = 0 to 700

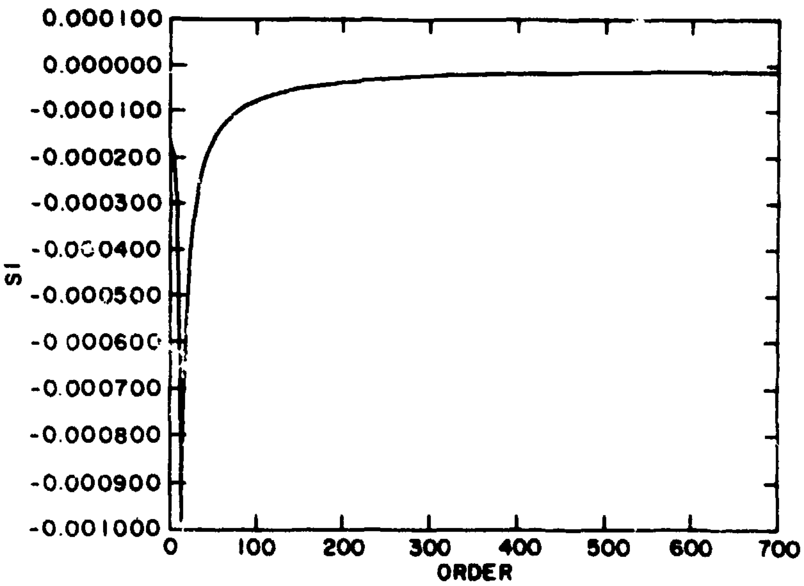


Figure 9. SI(n) = $\int Q(\alpha)$ ImFS(n, α) for n = 0 to 700

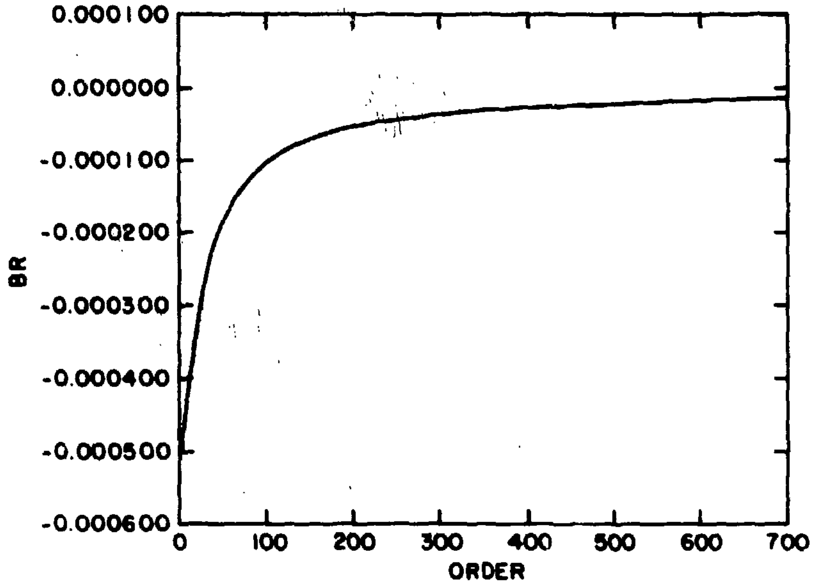


Figure 10. $BR(n) = \int Q(\alpha)\, ReFB(n, \alpha)$ for n = 0 to 700

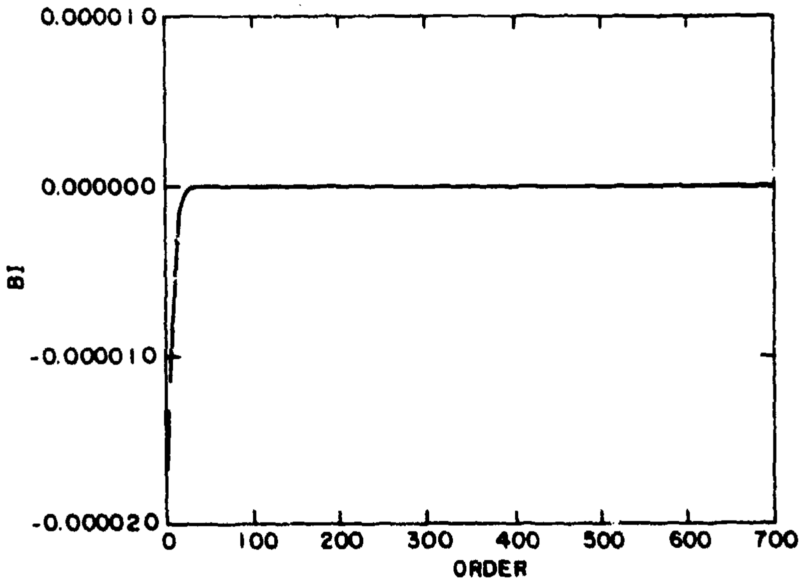


Figure 11. $BI(n) = \int Q(\alpha)\, ImFB(n, \alpha)$ for n = 0 to 700

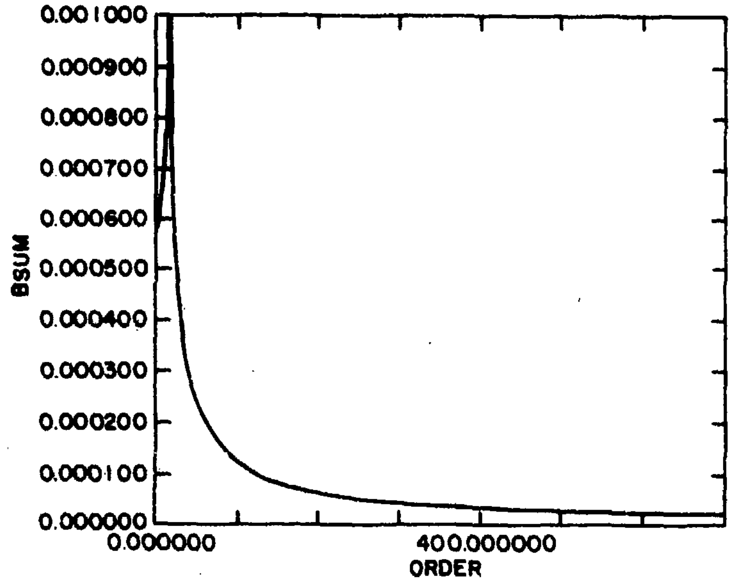


Figure 12. $BSUM(n) = \sqrt{(SR(n) + BR(n))^2 + (SI(n) + BI(n))^2} = |BESS(n)|$ for n = 0 to 700

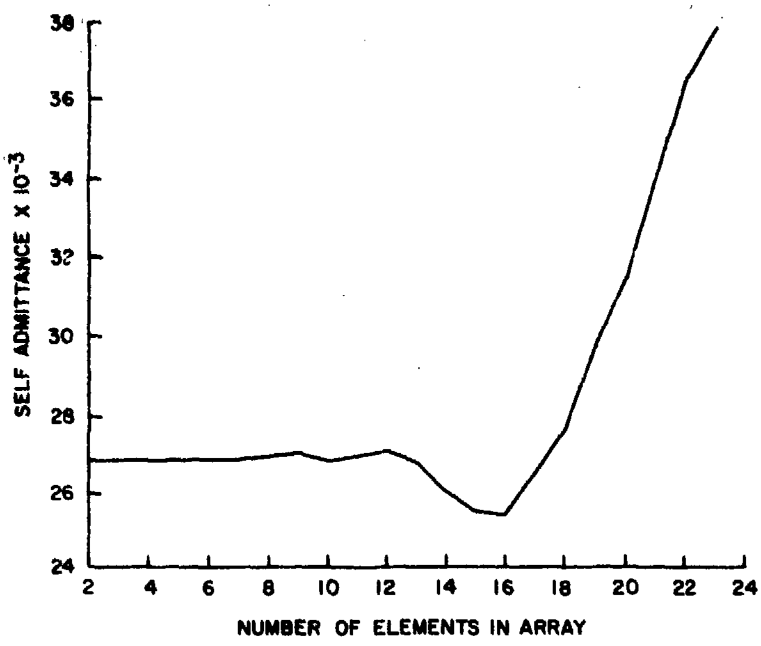


Figure 13. Self-Admittance for Active Element for Arrays of Size m = 2 to 22

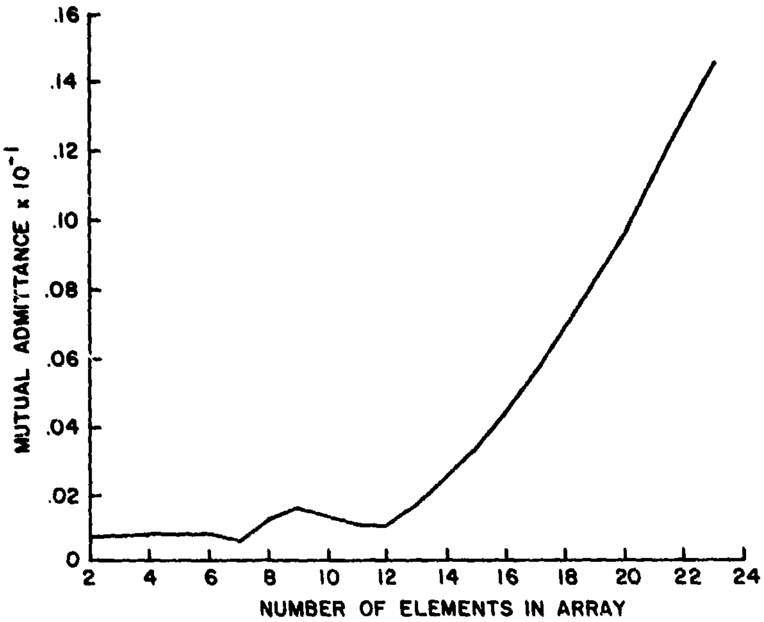


Figure 14. Mutual Admittance Between the Active Element (Dipole No. 1) and the Adjacent Dipole (No. 2)

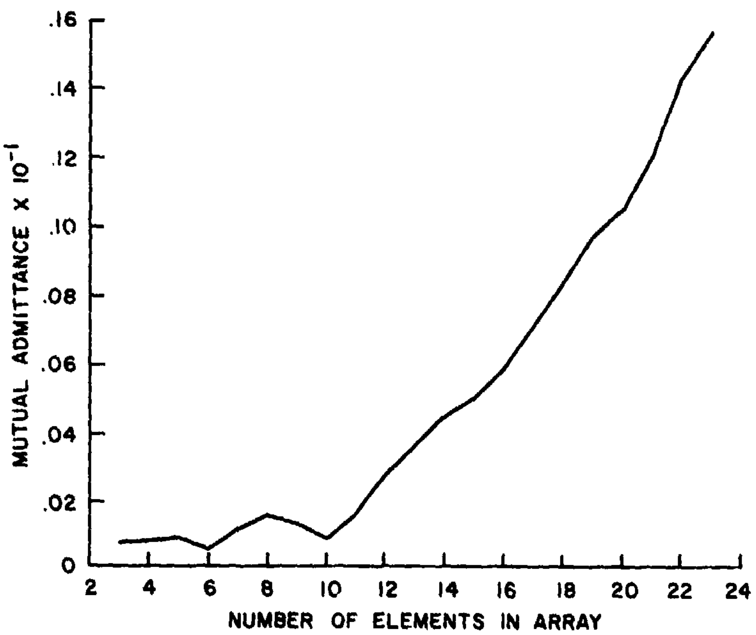
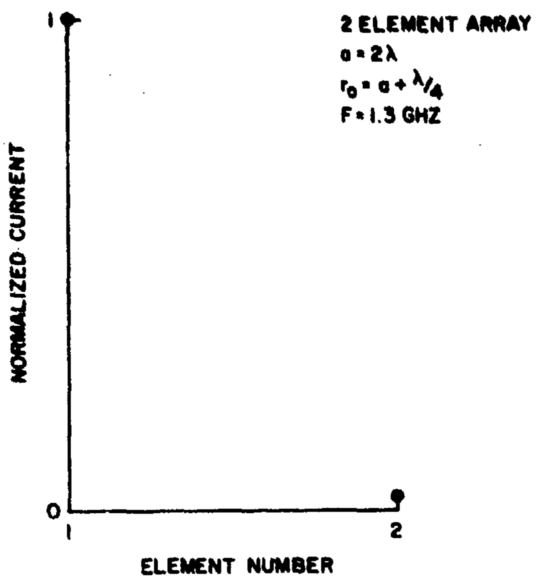


Figure 15. Mutual Admittance Between the Active Element (Dipole No. 1) and the Adjacent Dipole (No. m)



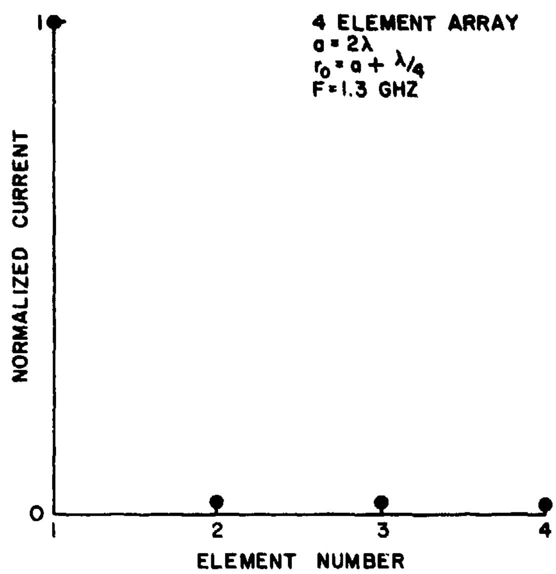
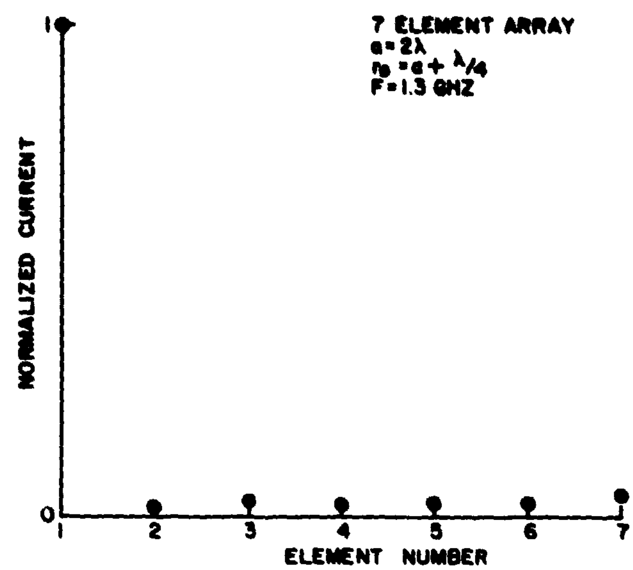


Figure 16. Relative Currents For the Circular Array of Dipoles Above a Perfectly Conducting Cylinder For Active Element Dipole No. 1, m = 2, 4, 7, 10, 13, 16



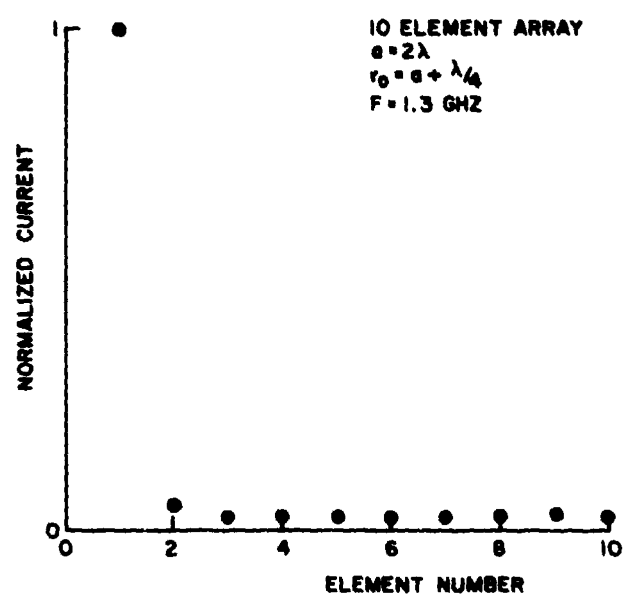
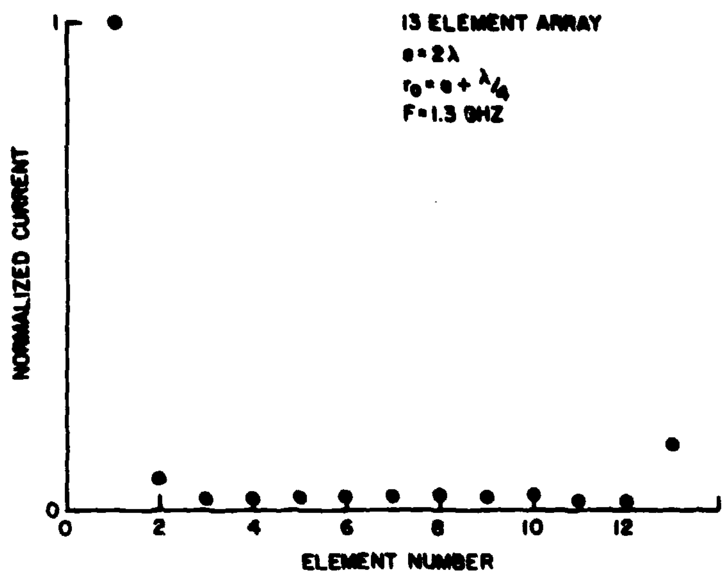


Figure 16. Relative Currents For the Circular Array of Dipoles Above a Perfectly Conducting Cylinder For Active Element Dipole No. 1, m = 2, 4, 7, 10, 13, 16 (Cont.)



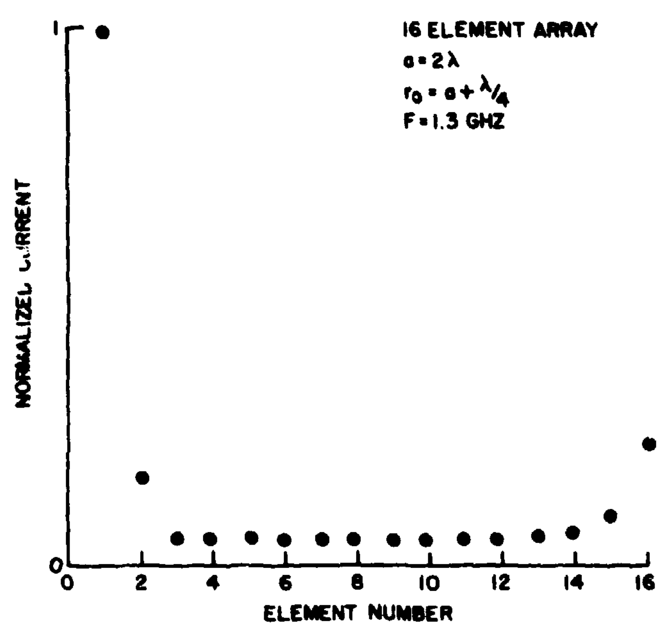


Figure 16. Relative Currents For the Circular Array of Dipoles Above a Perfectly Conducting Cylinder For Active Element Dipole No. 1, m = 2, 4, 7, 10, 13, 16 (Cont.)

3. CONCLUSION

It has been shown that numerical techniques for approximation of the solution of the m nonhomogeneous simultaneous equations (Eq. (37)) provide useful and analytical data in the determination of characteristics of a circular array above a perfectly conducting cylinder. From Eqs. (37) and (50), it is seen that the value BESS(n) must converge to zero for large order n to determine the coefficients of the A_m's in Eq. (39). Due to the symmetry of the coefficients, as seen in Eq. (51), the values T_{11}, T_{12}, ... T_m need only be calculated once for a given array of size m. Also, from the expression for BESS(n), Eq. (50), BESS(n) is independent of m, the number of dipole elements in the circular array being considered. BESS(n) is, however, dependent on the values of the radius of the cylinder and the frequency (Wavelength). Consequently, tables can be made of the BESS(n) values for n = 0 to N, where N is of order n large enough to force BESS(n) to zero. These tables would enable calculation of the mutual and self admittances for an array of arbitrary size m about a perfectly conducting cylinder of radius r'. An understanding of the effects of mutual coupling, taking into consideration both the space wave and the creeping wave, is necessary in the synthesizing of an aperture distribution to give the best fit to a specified radiation pattern, and for the determination of optimization techniques for circular array antennas.

References

1. Provencher, J.H. (1970) A survey of circular symmetric arrays, Phased Array Antennas, (Oliver and Knittel, Artech House, Inc., Dedham, Mass.) page 292.

2. Hessel, A. (1970) Mutual coupling effects in circular arrays on cylindrical surfaces—aperture design implications and analysis, Phased Array Antennas, (Oliver and Knittel, Artech House, Inc., Dedham, Mass.) page 273.

3. Carter, P.S. (1943) Antenna arrays around cylinders, Proceedings of IRE, Vol. 31, December.

4. Lucke, W. (1949) Electric Dipoles in the Presence of Elliptical and Circular Cylinders, Report No. 1, Project 188, Stanford Research Institute, Stanford, Calif.

5. Lucke, W. (1951) Electric dipoles in the presence of elliptical and circular cylinders, Journal of Applied Physics, Vol. 22, No. 1, January.

6. Harrington, R.F., and LePage, W.R. (1949) A Study of Directional Antennas for DIF Purposes, Report 1, Department of Electrical Engineering, Syracuse University, Syracuse, NY, September.

7. Harrington, R.F. (1961) Time-Harmonic Electromagnetic Fields, McGraw-Hill, New York, NY, Chapter 3.

8. Neff, H.P., Hickman, C.E., and Tillman, J.D. (1964) Circular Arrays around Cylinders, Report No. 7, Department of Electrical Engineering, University of Tennessee, Contract AF19(628)-288, June.

9. Fante, Dr. Ronald L. (private communication).

10. Tillman, James D. (1966) The Theory and Design of Circular Antenna Arrays, Chapter 1, The University of Tennessee, Engineering Experiment Station.

11. Amos, V.E., and Daniel, S.L. (1977) AMOSLIB, A Special Library Version, Sandia Laboratories (SAND 77-1390).

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